The mantle, the core and magma oceans in the thermal evolution of the Earth

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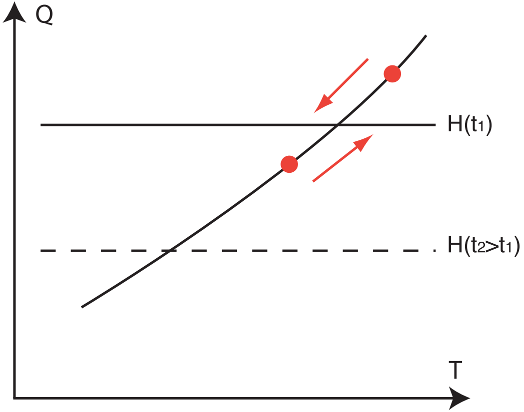
FDEPS, Nov, 2019

Principle for thermal evolution models



- Global energy balance

$$MC_P\frac{dT}{dt} = -Q(T) + H(t)$$

- Evolution time scale

$$\tau = \frac{MC_P T}{Q(T)}$$

- The Urey number (time dependent)

$$Ur = 100\frac{H}{Q}$$

- Question:
  - what is the function $Q(T)$ for mantle convection?

Boundary layer scaling for thermal evolution model

- Boundary layer scaling

$$\Rightarrow q = C\frac{k\Delta T}{d} Ra^{1/3}\left(\frac{T_i}{\Delta T}\right)^{4/3}$$

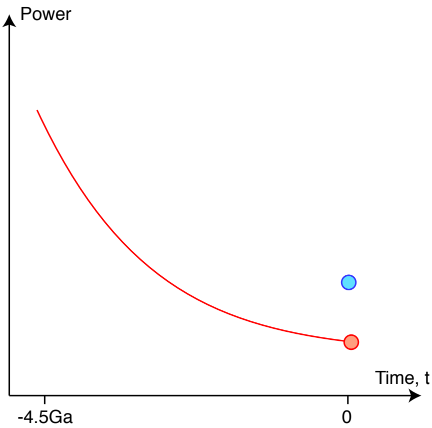
- Assuming the scaling applies to mantle convection, hence to the present time ($t = 0$): Q_0, T_0.
- Scaling of surface heat flow:

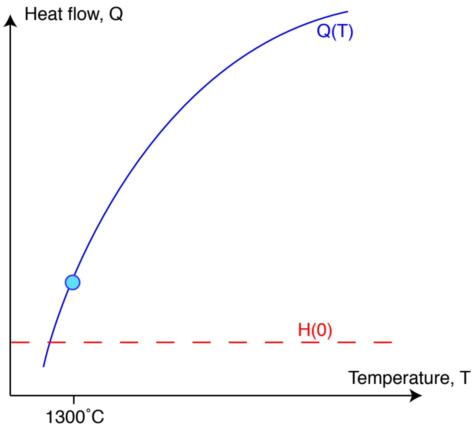
$$Q(T) = Q_0\left(\frac{T}{T_0}\right)^{4/3}\left(\frac{\eta(T)}{\eta_0}\right)^{-1/3}$$

- Given the present-day Urey number: $Ur = 100 \times H/Q \in [20\% - 50\%]$.
- Solve the energy equation backward from the present time:

$$MC\frac{dT}{dt} = H(t) - Q(T)$$

The low Urey number "paradox"





$$MC\frac{dT}{dt} = H(t) - Q(T); \qquad Q(T) = Q_0 \left(\frac{T}{T_0}\right)^{4/3} \left(\frac{\eta(T)}{\eta_0}\right)^{-1/3}$$

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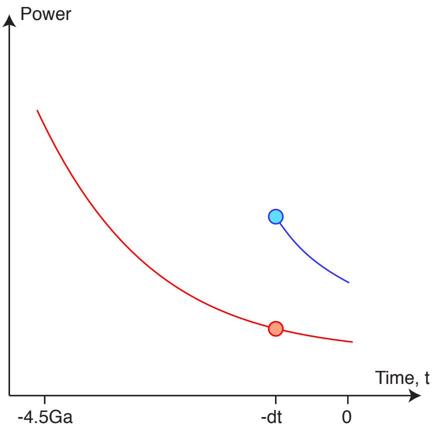

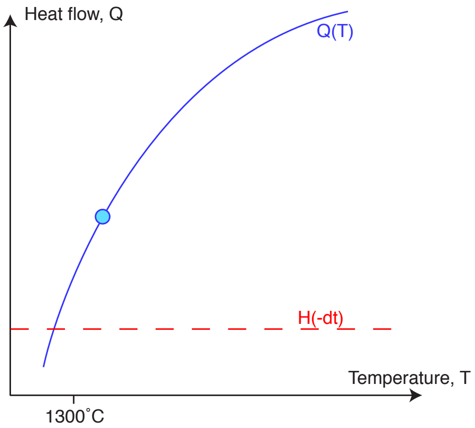

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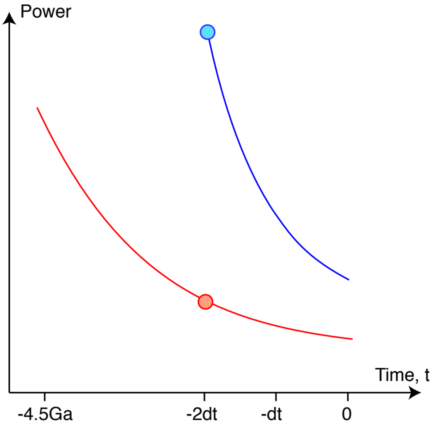

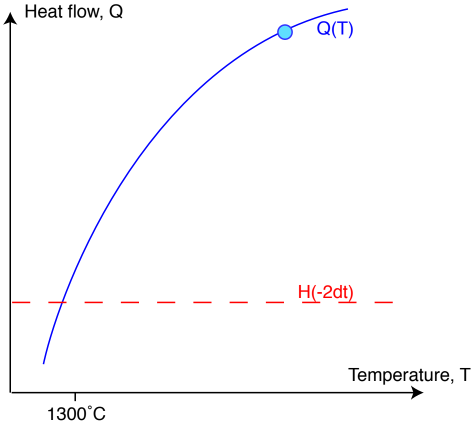

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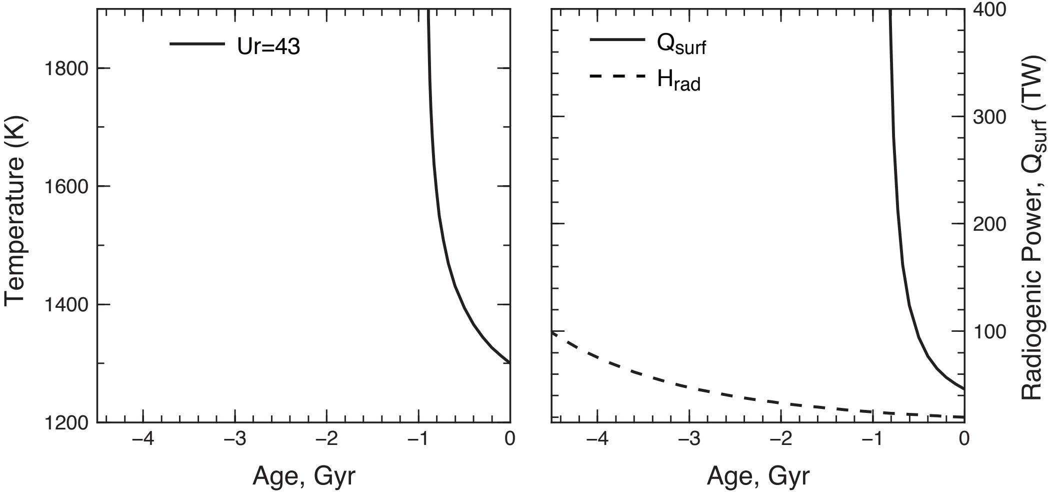
Feedback with temperature-dependent viscosity



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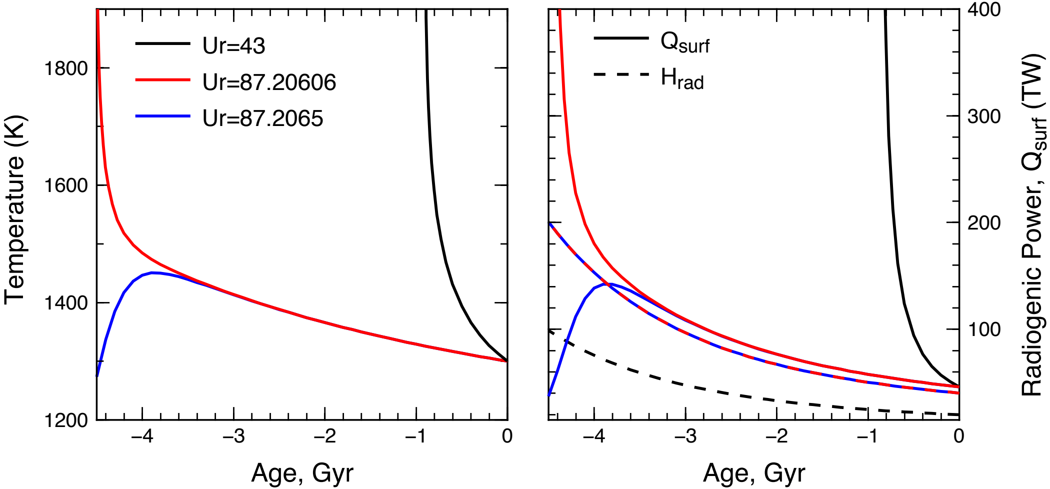

Proposed solutions

- Layered mantle convection $\Longrightarrow$ lower cooling efficiency (McKenzie & Richter, 1981). But the inferred lower mantle temperature is too large to keep it solid unless it is depleted in heat producing elements (Schubert & Spohn, 1981; Spohn & Schubert, 1982).
- A smaller exponent β in the $q = ARa^{\beta}$ scaling law (Christensen, 1985; Conrad & Hager, 1999; Sleep, 2000; Korenaga, 2003).
- Differential core-mantle cooling (another type of layered model).

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A lower β exponent to decrease the feedback?

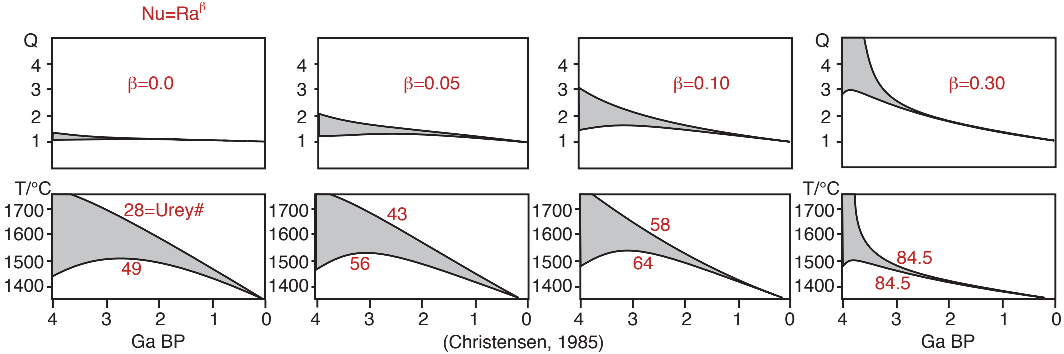
Christensen (1985):

> local resisting forces in the subduction region. The cause for this resistance may be (1) shear on the thrust fault toward the overriding plate, (2) resistance against the bending of the downgoing plate, (3) resistance against penetration into a high viscosity layer at greater depth, (4) resistance against bending or viscous deformation at the boundary between upper and lower mantle, or (5) resistance against penetration through an endothermic phase boundary [*Christensen and Yuen*, 1984]. In cases 1, 2, and 5 the resisting force would be entirely independent from the asthenospheric temperature and viscosity, possibly also in case 4. The compressive stress which is usually

$\Rightarrow$ A decreased feedback between the mantle temperature and the surface heat flow.

Effect of the β exponent on thermal evolution

$$Q(T) = Q_0 \left(\frac{T}{T_0}\right)^{1+\beta} \left(\frac{\eta(T)}{\eta_0}\right)^{-\beta}$$



Same solution was advocated by Conrad & Hager (1999, 2001), Sleep (2000), Korenaga (2003).

Problem: No self-consistent dynamical model gives such low values of β

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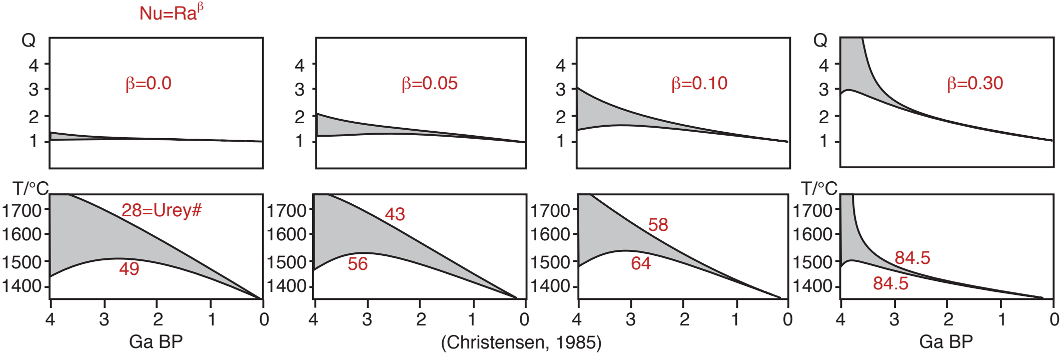

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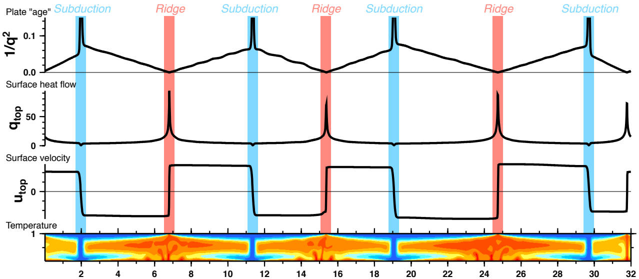
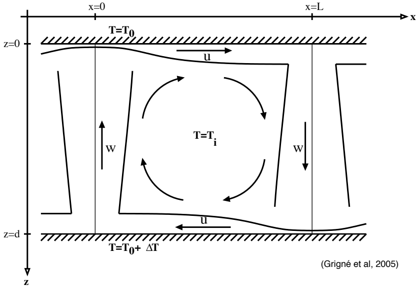
Heat flow and plate size



(Grigné et al, 2005 ; Labrosse & Jaupart, 2007)



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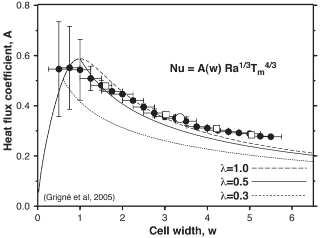

- Loop model: balance between buoyancy and viscous resistance

$$\Rightarrow q_{top} = C(L) Ra_m^{1/3} T_m^{4/3}$$

$\Rightarrow$ classical scaling supported by convection models with self-consistent plate tectonics (pseudo-plastic rheology).

- Standard approach:

$$MC\frac{dT}{dt} = H(t) - Q(T)$$

parameterised by the mantle potential temperature only.

$\Rightarrow$ Core and mantle assumed to cool at the same pace.

- Assume instead that the core is cooling and not the mantle:

$\Rightarrow$ No feedback from temperature dependence of the mantle viscosity!

$$M_M C_M \frac{dT_M}{dt} = H(t) - Q(T_M) + Q_{CMB}$$

$$M_C C_C \frac{dT_C}{dt} = -Q_{CMB}$$

Alternative scenario

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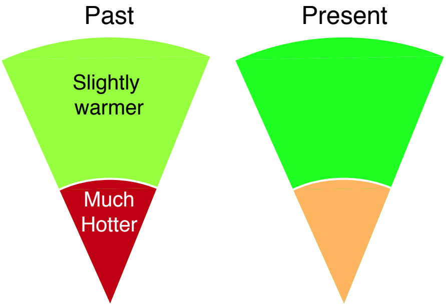

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$$M_C C_C \frac{dT_C}{dt} = -Q_{CMB}$$

A modified Urey number

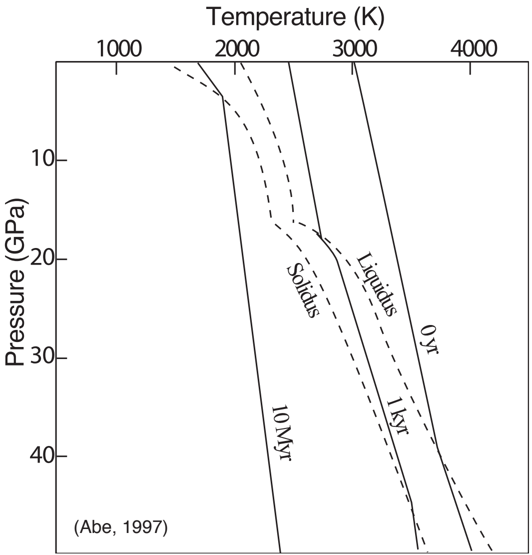
Labrosse, in AGU Mon. 207, 2016

- Core heat flow is a heat source to the mantle. Assume $Q_{CMB} = 13.1\,\mathrm{TW}$ (Buffett, yesterday, very precise).

⇒ Modified Urey number:

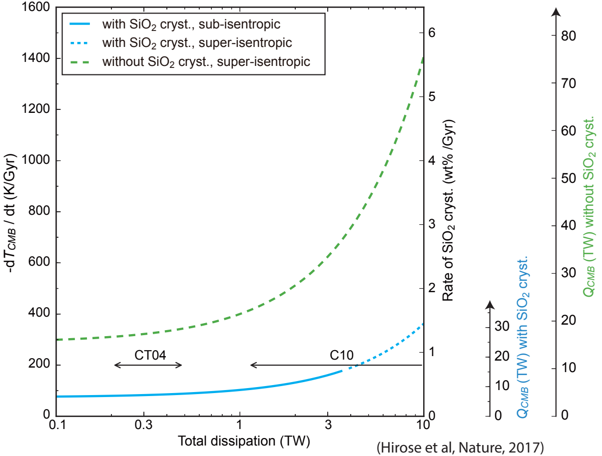
$$Ur^{\star} = \frac{H + Q_{CMB}}{Q_{surface}} = \frac{13 + 13.1}{39} = 0.67$$

- Arguments for differential cooling of the core and mantle?



- Total mantle cooling in 4.5 Gyr constrained by the phase diagram of the upper mantle: $\Delta T_m < 200K$

Core cooling constrained by the need to sustain a geodynamo

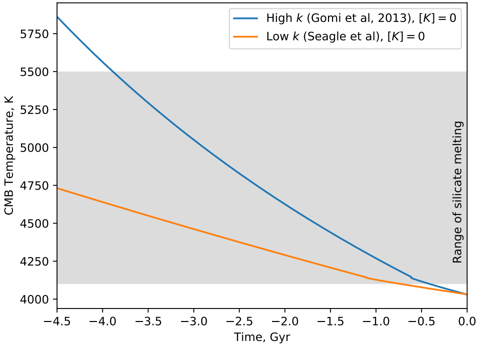


(Hirose et al, Nature, 2017)

Thermodynamics of a convective dynamo

- For a given dissipation (> 0), one can compute the CMB heat flow.
- Compute the cooling rate of the core
- CMB heat flow much smaller when compositional convection is active (IC growth and/or light element extraction)
- Results strongly depend on the value of the thermal conductivity (debated)

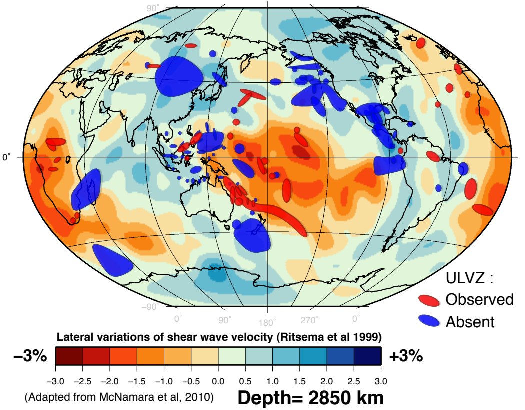
Example cooling histories



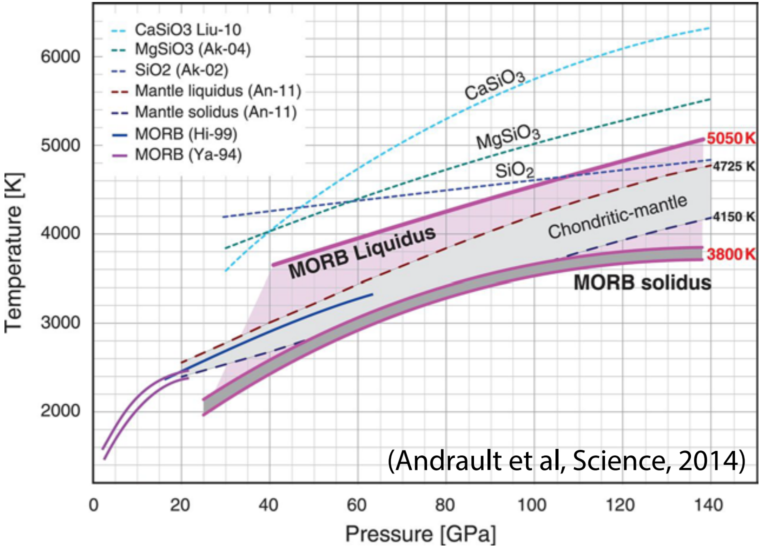
- Model with $Q_{CMB} = 1.15 Q_s$ at all time
- Current CMB temperature close to the solidus $\Longrightarrow$ lowermost mantle likely molten in the past.

Dense partial melt pocket at the base of the mantle

- Large V_S anomalies in the lower mantle $\rightarrow$ thermal and chemical heterogeneity.
- ULVZs at the edges of dense thermo-chemical piles. Interpreted as pockets of dense partial melt.

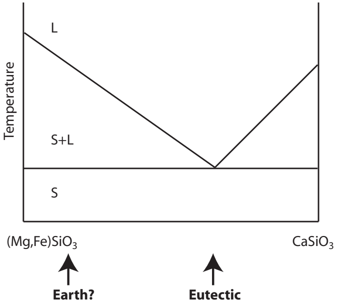


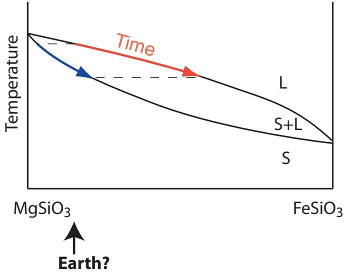
(Adapted from McNamara et al, 2010)



(Andrault et al, Science, 2014)

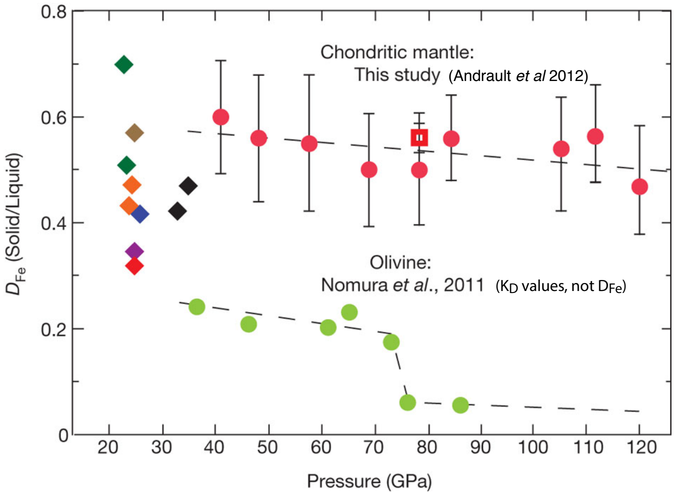
Ideas on melt processes





- For chemically complex systems, the composition of liquid and solid are different.
- In particular: Fe partitions preferentially in the liquid silicate rather than solid.
- Density change on melting: $\Delta\rho = \Delta\rho_\phi + \Delta\rho_\chi$.
  - $\Delta\rho_\phi$ decreases with pressure
  - $\Delta\rho_\chi < 0$

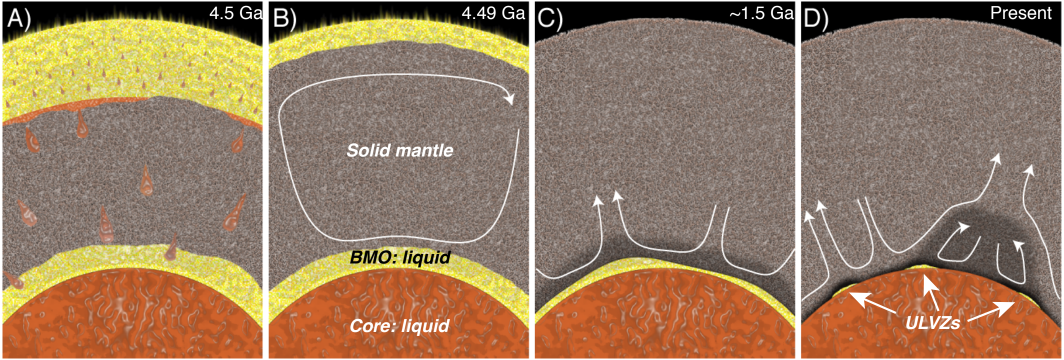
Composition of lower mantle melt



- Precise value of partition coefficient still debated but,
- Fe content of the liquid is larger than that of the solid

⇒ gets richer over time.

Crystallisation of a basal magma ocean (BMO)

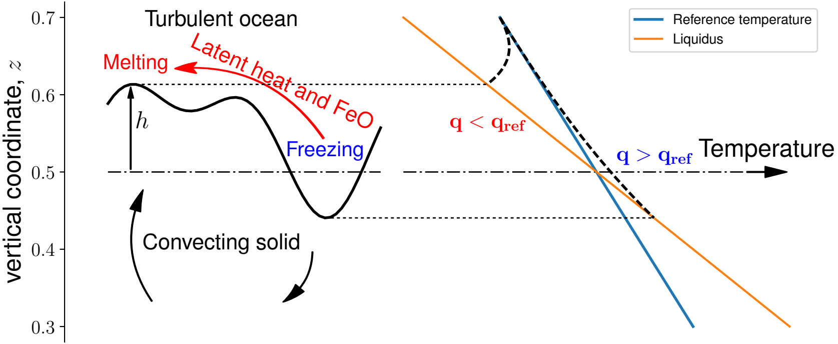


(Labrosse, Hernlund, Coltice, 2007)

- ULVZ: Dense partial melt at present
- Cooling of the core evidenced by the maintenance of the geodynamo for at least 3.5 Gyrs.
- $\Rightarrow$ More melt in the past!
- Fractional crystallisation $\Rightarrow$ compositional variations.

Phase change boundary conditions - 1

First developed for the inner core (Deguen, Alboussière, Cardin, et al)



- Viscous stress in the solid mantle $\Rightarrow$ topography builds with timescale $\tau_\eta = \frac{\eta}{\Delta \rho g d}$
- Heat transfer in the liquid erases topography with timescale $\tau_\phi = -\frac{\rho_s L}{\rho_l c_{pl} u_l \frac{\partial T_m}{\partial z}}$
- Competition of the two processes controlled by $\Phi = \frac{\tau_\phi}{\tau_\eta}$

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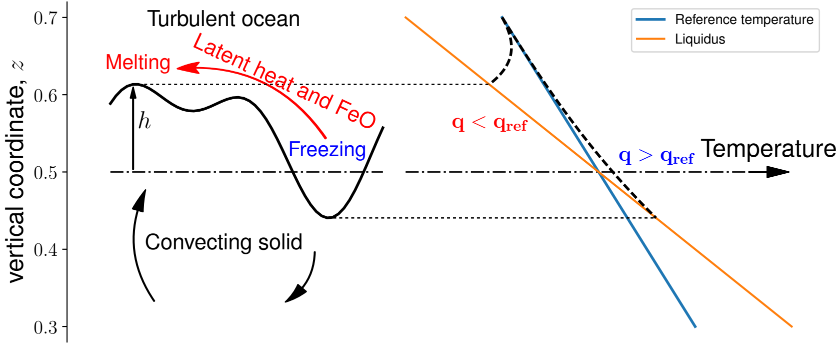

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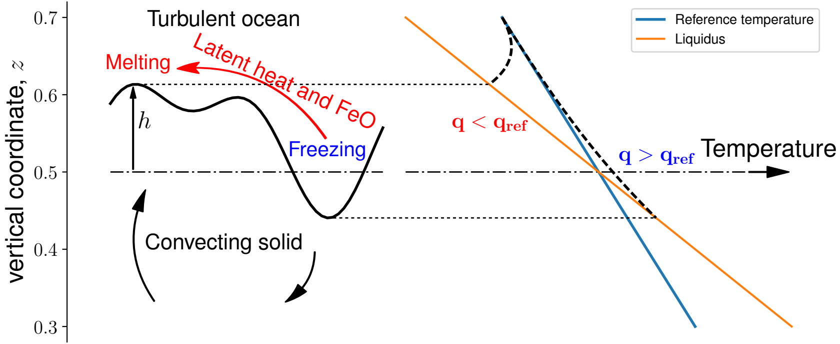

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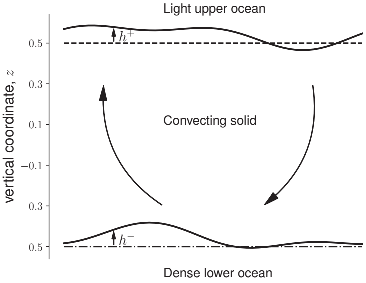
Phase change boundary conditions - 2



- Continuity of the normal stress:

$$-p + (\rho_s - \rho_l^{\pm})gh^{\pm} + 2\eta\frac{\partial w}{\partial z} = 0.$$

- Quasistatic relationship between the topography and the solid velocity, with τ_ϕ a phase change time scale:

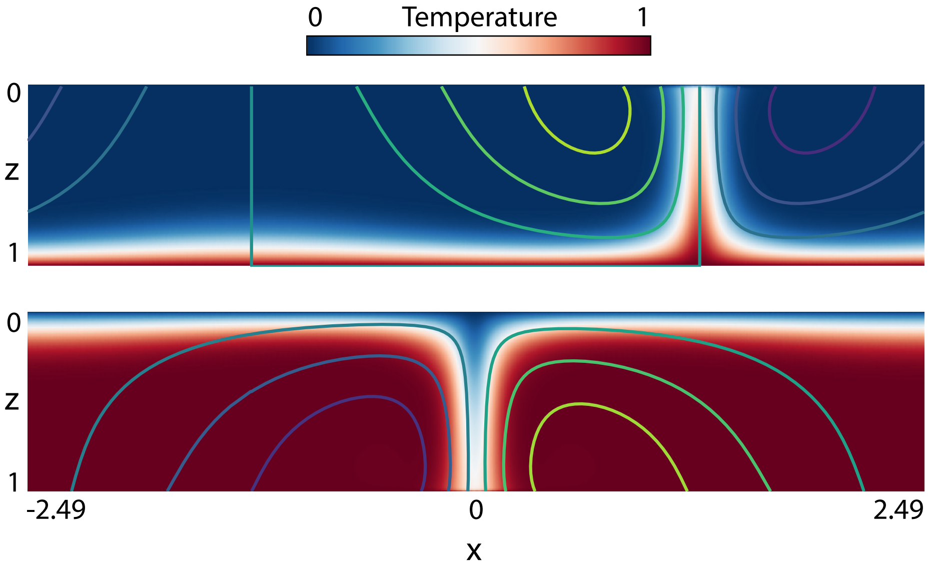
$$w = \frac{h^{\pm}}{\tau_\phi^{\pm}}$$

- Dimensionless boundary condition for vertical velocity:

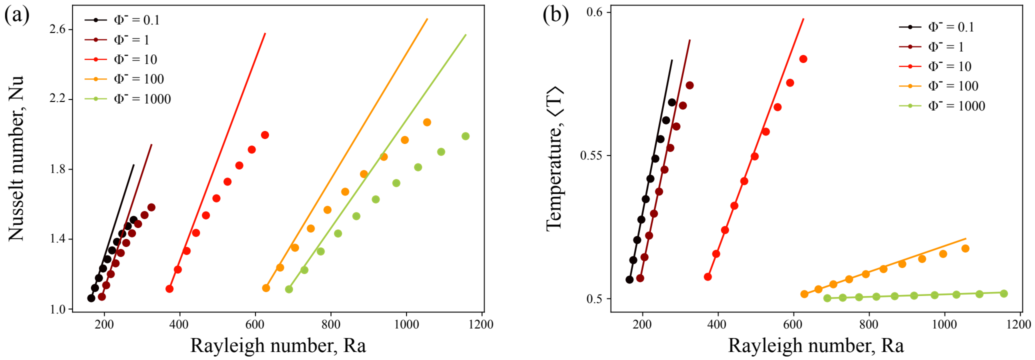
$$\pm\Phi^{\pm}w + 2\frac{\partial w}{\partial z} - p = 0, \qquad \text{with} \quad \Phi^{\pm} = \frac{\tau_{\phi^\pm}|\Delta\rho^{\pm}|gH}{\eta} = \frac{\tau_{\phi^\pm}}{\tau_\eta}$$

- $\Phi \to \infty \Rightarrow$ classical non-penetrative boundary condition ($w = 0$).
- $\Phi \to 0 \Rightarrow$ permeable boundary condition ($w \neq 0$).
- Also, continuity of horizontal stress and fixed (phase change) temperatures.

Thermal structure with one boundary with $\Phi = 0.1$

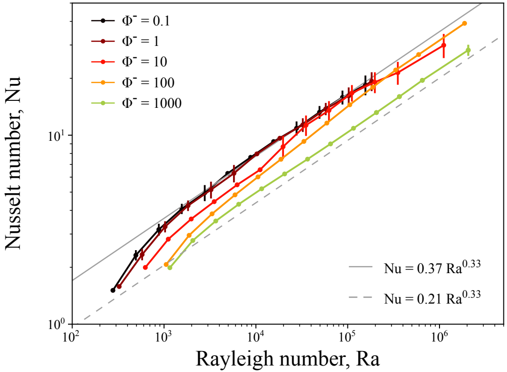


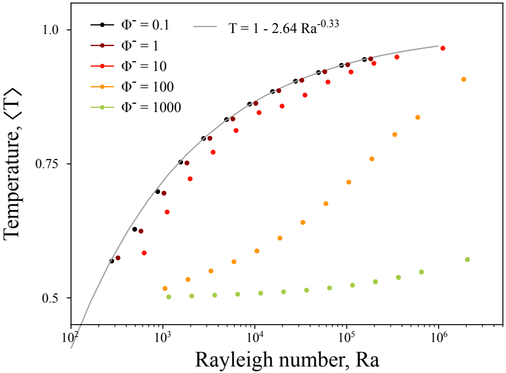
Heat transfer and mean temperature - close to onset



- Good match of the fully non–linear results (DNS) and the weakly non–linear ones for small Ra/Ra_c.
- Deviation at high Ra, more rapidly for heat flow (Nusselt number) than average temperature.

Heat transfer and mean temperature - high Rayleigh number



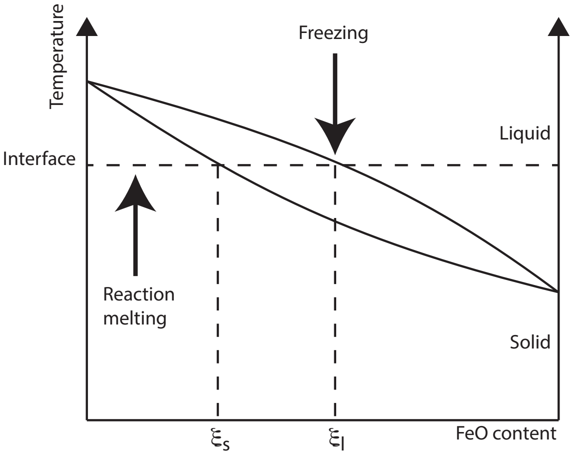


- At high Ra, $Nu \sim C Ra^{1/3}$.
- Coefficient C larger for small $\Phi \Rightarrow$ heat flow about twice larger for a given Ra.
- Consistent with a dynamics controlled by the only active boundary layer.

Toward a fully dynamical evolutionary model

- A joint effort:
  - Lyon: **A. Morison, R. Agrusta**, R. Deguen, T. Alboussière, S. Labrosse.
  - Zürich: **D. Bolrão**, A. Rozel, M. Ballmer, P. J. Tackley.
- Ingredients (within StagYY):
  - Convection in the solid mantle,
  - Tracers to treat composition (FeO content),
  - Moving boundary for the net motion of the interface,
  - Boundary condition to model phase change at the interface,
  - Compositional and thermal evolution of the BMO and core.

Composition: conditions for the phase change

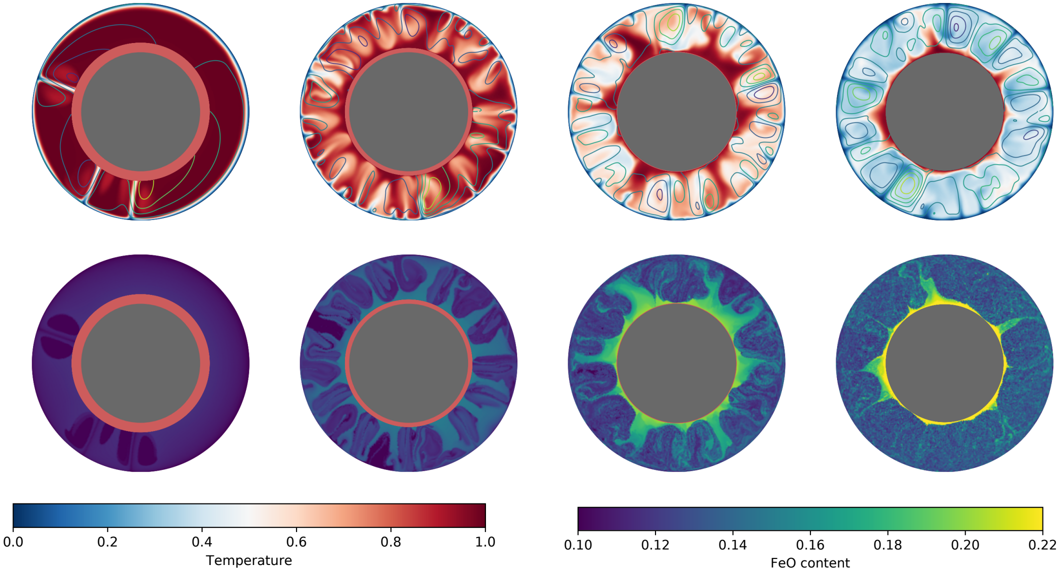


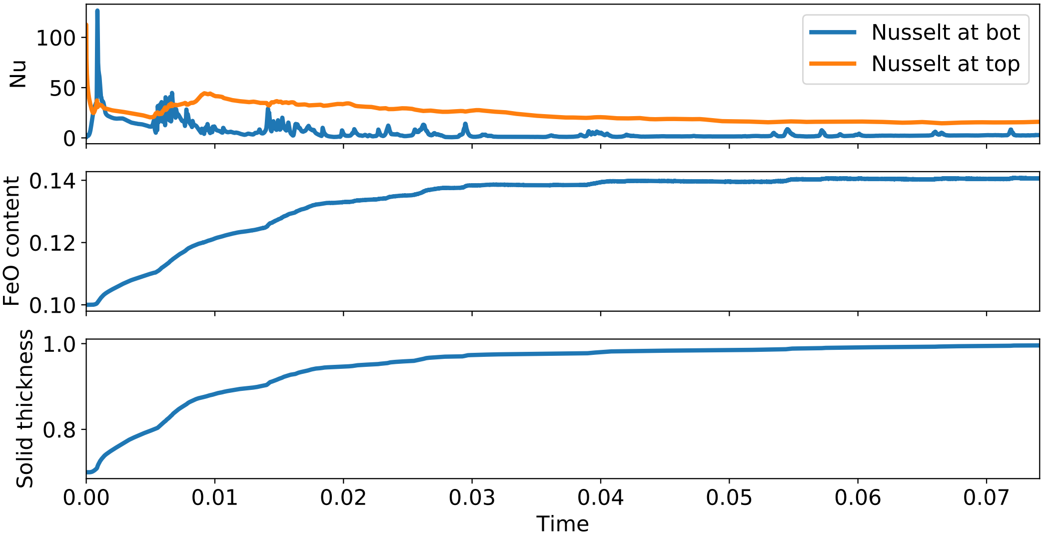
- Liquid of uniform composition ξ_l
- Freezes at $T_l(\xi_l) \Rightarrow$ solid of mass fraction $\xi_s = D\xi_l$.
- Liquid gets locally enriched.
- Solid with $\xi < \xi_s$ melts at $T < T_s(\xi_s)$ by reacting with the liquid.
- Needs FeO transfer in the magma ocean from regions of freezing to regions of melting.

Example of evolution

- Change of dynamical regime with time.
- Gradual stabilisation of a dense layer at the bottom.
- Timing strongly depends on parameters (Rayleigh number, Buoyancy number, rheology, internal heating, etc). To be explored further!

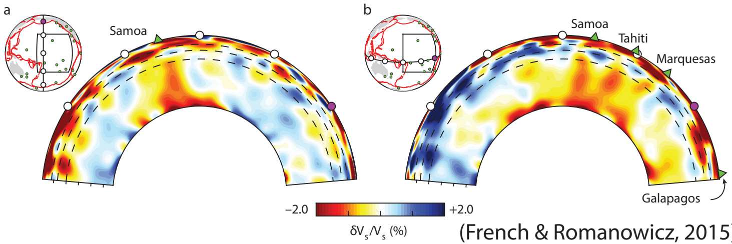
Preliminary simulation with crystallizing BMO



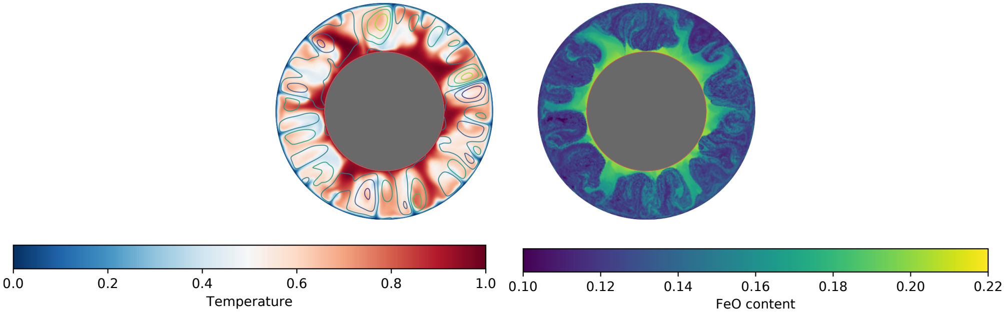


Nusselt at bot
Nusselt at top
Nu
0
50
100
FeO content
0.10
0.12
0.14
Solid thickness
0.8
1.0
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.07
Time

Explanation for broad plumes observed in the mantle?



(French & Romanowicz, 2015)



- The heat budget of the Earth is rather well established (with known uncertainties).
- Thermal evolution models can match the observation if exotic behavior is assumed for the mantle or (preferred) a core cooling faster than the mantle.
- Implies long-term persistence of a basal magma ocean.
- Convection in the solid greatly influenced by the possibility of melting and freezing at the bottom.
- Systematic exploration of the parameter space is needed: Rayleigh number, buoyancy ratio, rheology, internal heating, mineral phase diagram...

Conclusions

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